General Solution Example 2

Example 7: Find the general solution of the linear system in *parametric form.* Check your answer!

$$\begin{cases} x_1 + 2x_2 + x_3 + x_4 = 1 \\ 2x_1 + 3x_2 + x_3 + 2x_4 = 1 \end{cases} \tag{1}$$

$$\overset{x_1 \; x_2 \; x_3 \; x_4}{\begin{bmatrix} 1 & 2 & 1 & 1 & | & 1 \\ 2 & 3 & 1 & 2 & | & 1 \end{bmatrix}} \xrightarrow{R_2 := R_2 - 2R_1} \begin{bmatrix} 1 & 2 & 1 & 1 & | & 1 \\ 0 & -1 & -1 & 0 & | & -1 \end{bmatrix} \overset{R_2 := -R_2}{\xrightarrow{R_1 := R_1 + 2R_2}} \begin{bmatrix} 1 & 0 & -1 & 1 & | & -1 \\ 0 & 1 & 1 & 0 & | & 1 \end{bmatrix}$$

$$X_3 = S$$

$$X_4 = t$$

$$\begin{cases} X_1 - X_3 + X_4 = -1 & \Rightarrow & X_1 - S + t = -1 \Rightarrow X_1 = -1 + S - t \\ X_2 + X_3 \quad\quad = 1 & \Rightarrow & X_2 + S = 1 \Rightarrow X_2 = 1 - S \end{cases}$$

$$\boxed{\begin{array}{l} X_1 = -1 + S - t \\ X_2 = 1 - S \\ X_3 = S \\ X_4 = t \end{array}}$$

$S, t \in \mathbb{R}$

(s, t run through all real numbers)

check:

$(-1 + S - t) + 2(1 - S) + S + t = -1 + 2 = 1$

$2(-1 + S - t) + 3(1 - S) + S + 2t =$

$= -2 + 2S - 2t + 3 - 3S + S + 2t$

$= -2 + 3 = 1$

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